Thus, the parameters of a gas flow with solid particles in a nonsymmetric nozzle have been determined by using the proposed method. The calculation results explain the nonuniformity in the wear of the outlet section of a nonsymmetric nozzle.

NOTATION

x and y, present coordinates; u and u_s, projections of the velocities of the gaseous and the solid phases on the nozzle axis, respectively; v and v_s, projections of the velocities of the gaseous and solid phases on the normal to the nozzle axis, respectively; w and w_s, velocities of the gas and the particles, respectively; ρ, gas density; c, drag coefficient of a particle; m, particle mass, d, particle diameter; S, cross-sectional area of a particle; V, volumetric gas discharge; y_1, half of the dimension of the nozzle's outlet section; L_1 and L_2, curves of the nozzle profile; τ, time of particle motion; ν, kinematic viscosity coefficient of the gas; Δx_k, integration step; $\Delta\tau_k$, time of particle motion along the section Δx_k; h, solid particle trajectory. Indices: o, flow parameters at the nozzle inlet; s, solid phase; k, step number (k = 0, 1, 2, ...).

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FRICTION LOSSES ON END WALLS OF A VORTEX CHAMBER

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The first integrals are obtained by employing the Kantorovich method for the boundary-layer equations for a flow in a vortex chamber, and losses due to friction on the end walls are determined.

In [1] the distribution was investigated of velocities and pressures in a vortex chamber beyond the boundary-layer limits; in [2] its hydraulic characteristics were obtained without friction losses on the walls being taken into account. The experimental data of [3] show that the motion of a vertex flow near the end walls is accompanied by the formation of a radial current near the ends. To improve the design of devices with a vortex flow, the design being based on the solution of boundary-layer equations, friction losses are determined on the end walls as well as the velocity distribution in the boundary layer.

Let us consider the motion of a vortex flow in the zone of the main vortex (Fig. 1) for $r_1 < r < R_K$ [1]. Since the axial component of the velocity is u = 0, the initial system of equations is as follows:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = 0, \tag{1}$$

$$v\frac{\partial v}{\partial r} - \frac{w^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu_T\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial x^2}\right), \tag{2}$$

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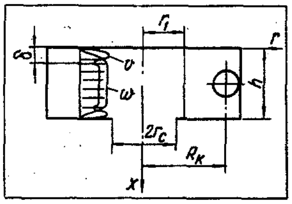

Fig. 3. Change in ψ as a function of r_c: 1) ψ 10^{-1}; 2) ψ ; 3) $\psi \cdot 10$.

$$v\frac{\partial w}{\partial r}+\frac{vw}{r}=\nu_T\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}+\frac{\partial^2 w}{\partial x^2}\right), \tag{3}$$

$$\frac{\partial v}{\partial r}+\frac{v}{r}=0. \tag{4}$$

In the above equations the kinematic-viscosity coefficient has been replaced by its turbulence analog. Since the satellite motion of the spirally twisted stream unavoidably results in the formation of vortical bunching with stabilized friction, one can therefore assume that ν_T = const for the entire flow including the boundary layer. Such an assumption was already made, for example, in [4, 5], and has been confirmed by experiments on the distribution of velocities in the boundary layer.

It follows from (1) that the pressure in the vortex chamber is constant with respect to the x coordinate. The radial distribution of pressure is now determined from (2) using (4):

$$\frac{1}{\rho}\frac{dp}{dr}=\frac{V^2+W^2}{r},$$

where the radial component of the velocity,

$$V=-\frac{|V_K|R_K}{r}$$

and the circular one,

$$W=W_K\left(\frac{r}{R_K}\right)^{1-k},$$

in the main flow were determined in [1].

By using the radial distribution of pressure Eqs. (2) and (3) are modified and are now

$$-\frac{v^2+w^2}{r}=-\frac{V^2+W^2}{r}+\nu_T\frac{\partial^2 v}{\partial x^2}, \tag{5}$$

$$v\frac{\partial w}{\partial r}+v\frac{w}{r}=\nu_T\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}+\frac{\partial^2 w}{\partial x^2}\right). \tag{6}$$

One has the following boundary conditions:

$$\text{for } x=0 \ v=0,\ w=0;$$
$$\text{for } x=\delta \ v=V,\ w=W,\ \frac{\partial v}{\partial x}=\frac{\partial w}{\partial x}=0. \tag{7}$$

By omitting the bars over the variables the dimensionless variables are

$$\bar{r}=\frac{r}{R_K},\quad \bar{x}=\frac{x}{R_K},\quad \bar{v}=\frac{v}{|V_K|},\quad \bar{w}=\frac{w}{W_K};$$

one writes now (5) and (6) as follows:

$$\frac{\partial^2 v}{\partial x^2}=\frac{k}{r}\left[\frac{1}{r^2}+B^2r^{2-2k}-(v^2+B^2w^2)\right], \tag{8}$$

$$\frac{\partial^2 w}{\partial x^2}=-\frac{\partial^2 w}{\partial r^2}+\left(kv-\frac{1}{r}\right)\frac{\partial w}{\partial r}+\left(kv+\frac{1}{r}\right)\frac{w}{r}, \tag{9}$$

where the value

$$B=\frac{W_K}{|V_K|} \tag{10}$$

corresponds to the tangent of the slope angle between the initial velocity and the radius.

It follows from the analysis of the experimental results in [3] that in the zone of the main vortex the profiles of the radial and of the circular components of velocity in the boundary layer show an affine similarity. Therefore, an approximate distribution of velocities is sought in the form

$$v=\varphi(x)\frac{1}{r}, \tag{11}$$

which is a solution of Eq. (4), and one also has

$$w=\Phi(x)r^{1-k}, \tag{12}$$

since the flow is self-similar. These functions satisfy the imposed boundary conditions.

Then (8) and (9) are transformed as follows:

$$\varphi''=\frac{k}{r^2}[(1-\varphi^2)+B^2r^{2(2-k)}(1-\Phi^2)], \tag{13}$$

$$\Phi''=\frac{k(2-k)}{r^2}\Phi(1-\varphi). \tag{14}$$

Integrating Eqs. (13) and (14) by using the method of Kantorovich [6] one obtains a system of ordinary differential equations of the second order,

$$\varphi''+a_1\varphi^2+a_2\Phi^2+a_3=0, \tag{15}$$

$$\Phi''+a_4\Phi(1+\varphi)=0, \tag{16}$$

where

$$a_1=\frac{k}{2\ln\frac{R_K}{r_1}}\left(\frac{R_K^2}{r_1^2}-1\right);\quad a_2=\frac{k}{k-1}\,\frac{B^2}{2\ln\frac{R_K}{r_1}}\left[\left(\frac{R_K}{r_1}\right)^{2(k-1)}-1\right];$$

$$a_3=-a_1-a_2,\quad a_4=(2-k)^2\frac{1-\left(\frac{R_K}{r_1}\right)^k}{1-\left(\frac{R_K}{r_1}\right)^{k-2}}.$$

The boundary conditions now are as follows:

$$\varphi(0)=0,\ \Phi(0)=0;\ \varphi(\delta)=-1,\ \Phi(\delta)=1,\ \varphi'(\delta)=\Phi'(\delta)=0. \tag{17}$$

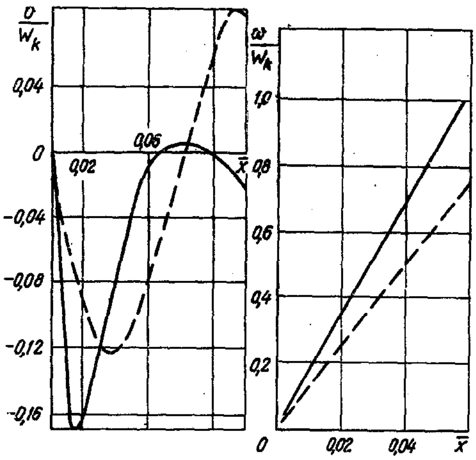

Fig. 2

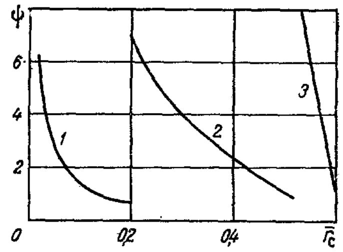

Fig. 3

Fig. 2. Distribution of radial and circular velocity components in a boundary layer (continuous curves – experimental data [3]; dashed curves – design data).

Fig. 3. Change in ψ as a function of $\bar{r}_c$: 1) $\psi \cdot 10^{-1}$; 2) ψ ; 3) $\psi \cdot 10$.

The solution of this two-point boundary-value problem is a function of the thickness of the boundary layer; it was obtained by using a digital computer, the sequence being as follows. First, the minimum is found for the functions $\varphi[\delta, \varphi'(0), \Phi'(0)] + 1$, $\Phi[\delta, \varphi'(0), \Phi'(0)] - 1$, then having obtained the values of $\varphi'(0)$ and $\Phi'(0)$ one determines the velocity distribution by using the Runge–Kutta method. The results of the computations for a vortex chamber investigated for a gas in [3] are shown in Fig. 2 and are in good agreement with the experiment. The assumption is thus confirmed that the turbulence analog of the kinematic-viscosity coefficient is constant in the case of a flow in the vortex chamber including the next to the wall zone.

To find the first integrals of the equations, new variables are introduced:

$$\varphi' = p(\varphi),\ \varphi'' = pp',\ \Phi' = q(\Phi),\ \Phi'' = qq', \tag{18}$$

which enable us to lower the order of the system,

$$pp' + a_1\varphi^2 + a_2\Phi^2 + a_3 = 0,\ qq' + a_4(1+\varphi)\Phi = 0. \tag{19}$$

Equations (19) with separated variables can be integrated. Then

$$\frac{p^2}{2} = -\frac{a_1}{3}\varphi^3 - (a_2\Phi^2 + a_3)\varphi + C_1,\ q^2 = -a_4(1+\varphi)\Phi^2 + C_2. \tag{20}$$

From the boundary conditions (17) we find for the flow core $C_1 = (2/3)a_1$, $C_2 = 0$.

If one now returns to the original variables and uses the boundary conditions one obtains the values of the derivatives at the end wall,

$$\varphi'(0) = \sqrt{\frac{4}{3}a_1},\quad \Phi'(0) = 0. \tag{21}$$

Thus the first integrals have been obtained by setting the derivatives equal to zero at the outer boundary as if the system had been overdetermined by them. Since the derivatives of velocities with respect to the coordinate x are known, one can solve the problem of friction losses on the end walls. The friction force on the two opposite walls in the zone of the main vortex are found in terms of the tangential tension,

$$F_{\text{fric}} = 2 \int_{(S)} \tau_n dS,$$

where

$$\tau_n = \rho \nu_{\text{T}} \sqrt{\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2}\Bigg|_{x=0}.$$

Since

$$\frac{\partial v}{\partial x} = \frac{|V_K|}{r} \varphi'(x), \quad \frac{\partial w}{\partial x} = \frac{W_K}{r^{k-1}} \Phi'(x),$$

therefore by using (21) one finds that

$$\tau_n = \rho \nu_{\text{T}} \sqrt{\frac{4}{3} a_1} \frac{|V_K|}{r}.$$

The vanishing of the circular component of the tangential tension $\tau_{x\varphi} = 0$ and of the moment of momentum $M_{x\varphi} = 0$, respectively, points to the fact that the mixing in the flow core is decisive for losses of the moment of the original fluid vortex in the main vortex zone.

Integrating over the surface from r_1 to R_K one finds

$$F_{\text{fric}} = 8\pi \rho \nu_{\text{T}} R_K |V_K| \left(1 - \frac{r_1}{R_K}\right) \sqrt{\frac{a_1}{3}}.$$

To evaluate the pressure losses due to friction when determining the hydraulic characteristics of the vortex chamber it is advisable to employ the ratio of the frictional force to the velocity head at the outlet of the nozzle,

$$\xi_A = \frac{F_{\text{тр}}}{\frac{\rho}{2}\left(\frac{Q}{\pi r_c^2}\right)^2 \pi r_c^2}.$$

In accordance with [1], if r_1/R_K is replaced by r_c/R_K for the values $r_c/R_K < 0.6$, one finally obtains

$$\xi_A = \frac{\psi}{\sqrt{k}} \left(\frac{r_c}{h}\right)^2.$$

The correcting function

$$\psi = \sqrt{8 \frac{\omega_1^2 - 1}{\ln \omega_1}} \left(\frac{1}{\omega_1} - 1\right),$$

where

$$\omega_1 = \sum_{n=1}^{\infty} \left(a \frac{r_c}{R_K}\right)^n \frac{1}{n!},$$

is shown in Fig. 3. It follows from the formulas that the fluid friction on the end walls of a vortex chamber with $h > 2r_c$ has little effect on its hydraulic characteristics.

To give an example it can be shown that for a vortex chamber investigated in [7] with R_K = 12.5 mm, r_c = 1.6 mm, h = 15 mm, k = 1.8 the quantity ξ_A = 0.187 is ∿40 times smaller than the overall resistance coefficient ζ = 7.95 [2].

NOTATION

r, x, φ, coordinates; v, w; V, W, radial and circular velocity components for the boundary layer or the main flow, respectively; p, pressure; ρ, density; ν_T, turbulence analog of kinematic viscosity coefficient; δ, thickness of boundary layer.

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EFFECT OF RADIATION ON THE SUPERSONIC FLOW OF A VISCOUS IONIZED GAS PAST BLUNT BODIES

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The supersonic flow of a viscous monatomic ionized gas past blunt bodies is investigated. The effect of radiation on the field of flow and on the heat flux transmitted to the wall is shown.

When a gas flows past a blunt body at supersonic speed, the presence of the high temperatures that arise in the wake of a shock wave leads to changes in the physicochemical properties of the gas because there is excitation of the internal degrees of freedom of the molecules, dissociation, ionization, and radiation. Depending on whether the time taken by these processes is comparable to the characteristic time of flow in the shock layer or is much shorter, the conditions of flow past the body will be nonequilibrium or equilibrium conditions. In the first case we must consider the actual kinetics of the nonequilibrium processes.

The flow of a monatomic nonequilibrium-ionized radiating gas in a shock layer was considered in [1-4], but only in the ideal-gas model. In [5] the case of flow of a viscous nonequilibrium-ionized gas was analyzed without taking account of radiation.

In the present article we investigate the flow of argon past blunt bodies, with the following ionization reactions taking place in the gas:

$$A + M \rightleftarrows A^* + M, \quad A^* + M \rightleftarrows A^+ + M + e, \tag{1}$$

$$A + h\nu \rightleftarrows A^+ + e, \tag{2}$$

where A, A* denote the atom in the ground state and an excited state; A^+ denotes a singly charged ion; e denotes an electron; hν denotes a photon; M is A or e.

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